Lepton Flavour Violation in Unparticle Physics

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Abstract

Recently H. Georgi has introduced an unparticle $\mathcal{U}$ in order to describe the low energy physics of a nontrivial scale invariant sector of an effective theory. In this work we have explored the phenomenology of an unparticle using the lepton flavour violating decay $\mu^- \to e^- e^+ e^-$, and found that the branching ratio of this decay is strongly dependent on the scaling dimension.

In particle physics the concept of scale invariance is very predictive in analysing the asymptotic behaviour of the correlation functions at high energies. The scale invariance at low energies is broken by the masses of the particles. However, at a scale much beyond the Standard Model (SM) the scale invariance is restored. Thus, if there is a scale invariant sector, it can be experimentally probed at TeV scale. Recently Georgi [1, 2] observed that such a conformal sector, due to the Banks-Zaks (BZ) fields [3], with scale dimensions $d_{\mathcal{U}}$, might appear at the TeV scale. This sector at low energies he termed as "unparticles" ($\mathcal{U}$) [1], which are invisible massless particles. In high energy collider experiments, like the LHC or ILC, the missing energy distributions of various processes can serve as a good test for the existence of these unparticles.

The phenomenological implications of these unparticles in the context of charged Higgs boson decays, anomalous magnetic moments, and meson anti-meson mixings has been studied in reference [4]. The possible effects of the $\mathcal{U}$ particle in the missing energy signatures of $Z \to q\bar{q}\mathcal{U}$, $e^+ e^- \to \gamma\mathcal{U}$, and the monojet production at hadronic collisions were performed in reference [5]. Note that there has also been another recent study of unparticle physics within the context of B-decays [6], where the effects of $\mathcal{U}$ in hadronic Flavour Changing Neutral Current (FCNC) processes was discussed. In another work the constraints on unparticle physics from electron (g-2) to positronium decays has been analysed [7]. In this work we have tried to explore the phenomenology of an unparticle ($\mathcal{U}$) in the lepton flavour violating decay $\mu^+ \to e^- e^+ e^-$.

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Before commencing with the calculation a few words on this theory are in order. At very high energy the theory contains the SM fields and the fields of the scale invariant sector, called BZ fields [3]. The BZ sector interacts with SM particles through the exchange of a connector sector which has an high mass scale $M_{\mathcal{U}}$. Below this mass scale the interaction between the SM and the BZ sector manifests itself in terms of non-renormalizable operators of the form;

$$O_{SM}O_{BZ}/M_{\mathcal{U}}^k \quad (k > 1) \ , \tag{1}$$

where O_{SM} and O_{BZ} are the local operators constructed from the SM and BZ fields. The renormalization effects in the scale invariant sector induce the dimensional transmutation at the scale $\Lambda_{\mathcal{U}}$ [8]. In the effective theory, below the scale $\Lambda_{\mathcal{U}}$, of the BZ operators match onto unparticle operators and non-renormalizable operators given by equation (1), where this matching gives rise to a new set of interactions, having the form;

$$\frac{C_{\mathcal{U}}\Lambda_{\mathcal{U}}^{d_{BZ}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}O_{SM}O_{\mathcal{U}} \ . \tag{2}$$

Note that d_{BZ} and $d_{\mathcal{U}}$ are the scaling dimensions of the O_{BZ} and unparticle $O_{\mathcal{U}}$ operators respectively, and the coefficient function $C_{\mathcal{U}}$ is fixed by the matching condition.

Three unparticles operators with different Lorentz structures; $O_{\mathcal{U}}$, $O_{\mathcal{U}}^{\mu}$ and $O_{\mathcal{U}}^{\mu\nu}$, are studied in reference [1]. The scale invariance was used to fix the two point correlation functions of these unparticle operators. Note that the abovementioned $O_{\mathcal{U}}^{\mu}$ and $O_{\mathcal{U}}^{\mu\nu}$ were taken to be transverse. In reference [2] the following set of effective operators, which can have interesting phenomenological implications, were given;

$$\lambda_0\frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}G_{\alpha\beta}G^{\alpha\beta}O_{\mathcal{U}} \ , \quad \lambda_1\frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\overline{f}\gamma_{\mu}fO_{\mathcal{U}}^{\mu} \ , \quad \lambda_2\frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}G_{\mu\alpha}G_{\nu}^{\alpha}O_{\mathcal{U}}^{\mu\nu} \ , etc.,$$

where $G^{\alpha\beta}$ is the gluon field strength tensor, f denotes SM fermions and λ_i are the dimensionless effective coupling constants with $\lambda_i = C_{O_{\mathcal{U}_i}^i}\frac{\Lambda_{\mathcal{U}}^k}{M_{\mathcal{U}}^k}$. Note that where we have used $i = 0, 1, 2$ we are referring to scalar, vector and tensor unparticle operators respectively. The implications of these operators were further analyzed in references [2, 5, 4]. In the analysis of unparticle physics in $e^+e^- \to \mu^+\mu^-$, by Georgi [2], these operators were *assumed* to be *"lepton-flavour-blind"*. In general these operators can be fermion flavour dependent. In reference [6] the operators were taken to be fermion flavour dependent and the analysis of these operators in various B-decays was performed.

For our study we will consider the following set of effective interactions for the unparticle operators which have couplings to leptons;

$$\frac{c_V^{\ell\ell'}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\overline{\ell}\gamma_{\mu}\ell'O_{\mathcal{U}} \quad + \quad \frac{c_A^{\ell\ell'}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\overline{\ell}\gamma_{\mu}\gamma_5\ell'O_{\mathcal{U}} \ . \tag{3}$$

The scalar operator couples with ordinary SM fermions and in principle can contribute to the problem under consideration. However, their contributions are proportional to the fermion mass
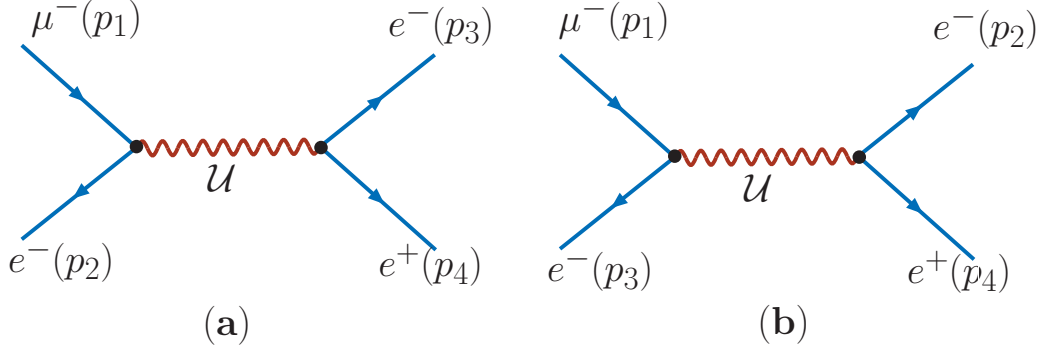
Figure 1: *Feynman Diagrams for* $\mu^- \to e^- e^+ e^-$

and are suppressed, as we have electrons (positrons) in the final state. For this reason we shall consider only the vector (axial-vector) unparticle operator in this paper.

The $\mu \to e^- e^+ e^-$ decay is described by the Feynman diagrams presented in figure (1). As such, in obtaining the matrix element for the $\mu^- \to e^- e^+ e^-$ decay we need to know the propagator of the vector unparticles. This propagator was first calculated in references [2, 5][§] and has the explicit form;

$$D_{\mu\nu}(x) = \int d^4x e^{iPx} < 0 \mid T[O_{\mathcal{U}}^\mu(x)O_{\mathcal{U}}^\nu(0)] \mid 0 >$$

$$= \frac{iA_{d_{\mathcal{U}}}}{2\sin d_{\mathcal{U}}\pi} \left(-g_{\mu\nu} + \frac{P^\mu P^\nu}{P^2}\right)(-P^2 - i\epsilon)^{d_{\mathcal{U}}-2} . \tag{4}$$

The vector operator is assumed to be transverse, that is $\partial_\mu O_{\mathcal{U}}^\mu = 0$, where

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})} . \tag{5}$$

After making these deductions we can write the matrix element for the $\mu^- \to e^- e^+ e^-$ decay as:

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 , \tag{6}$$

with

$$\mathcal{M}_1 = \bar{u}_2\gamma_\mu \left(a_1 + a_2\gamma_5\right) u_1 \bar{u}_3\gamma_\nu \left(a_3 + a_4\gamma_5\right) v_4 \ \frac{A_{d_{\mathcal{U}}}}{(\Lambda^{d_{\mathcal{U}}-1})^2} \frac{\left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{P^2}\right)}{2 \ sind_{\mathcal{U}}\pi}(-P^2 - i\epsilon)^{d_{\mathcal{U}}-2} , \tag{7}$$

$$\mathcal{M}_2 = -\bar{u}_3\gamma_\mu(a_1 + a_2\gamma_5)u_1\bar{u}_2\gamma_\nu(a_3 + a_4\gamma_5)v_4 \ \frac{A_{d_{\mathcal{U}}}}{(\Lambda^{d_{\mathcal{U}}-1})^2} \frac{\left(-g_{\mu\nu} + \frac{Q_\mu Q_\nu}{Q^2}\right)}{2 \ sind_{\mathcal{U}}\pi} \left(-Q^2 - i\epsilon\right)^{d_{\mathcal{U}}-2} , \tag{8}$$

where $P = p_1 - p_2 = p_3 + p_4$, $Q = p_1 - p_3 = p_2 + p_4$ and $a_1 = c_V^{\mu e}$, $a_2 = c_A^{\mu e}$, $a_3 = c_V^{ee}$, $a_4 = c_A^{ee}$.

[§]Both the papers, references [2] and [5], have given the explicit form of the propagator and appeared on the arXiv at the same time.

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For a massless electron and positron the matrix elements $\mathcal{M}_1$ and $\mathcal{M}_2$ simplify greatly, reducing to the following forms:

$$\mathcal{M}_1 = F_1 \, \overline{u}_2 \gamma_\mu (a_1 + a_2 \gamma_5) u_1 \overline{u}_3 \gamma^\mu (a_3 + a_4 \gamma_5) v_4 \ ,$$

$$\mathcal{M}_2 = -F_2 \, \overline{u}_3 \gamma_\mu (a_1 + a_2 \gamma_5) u_1 \overline{u}_2 \gamma^\mu (a_3 + a_4 \gamma_5) v_4 \ , \tag{9}$$

where

$$F_1 = \frac{A_{d_{\mathcal{U}}}}{(\Lambda^{d_{\mathcal{U}}-1})^2 \sin(d_{\mathcal{U}}\pi)} (-P^2 - i\epsilon)^{d_{\mathcal{U}}-2} \ ,$$

$$\text{and} \quad F_2 = F_1(P \to Q) \ . \tag{10}$$

The matrix element squared, for the considered process, is then:

$$|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + 2Re(\mathcal{M}_1^* \mathcal{M}_2) \tag{11}$$

with

$$|\mathcal{M}_1|^2 = 32|F_1|^2 [(a_1^2 + a_2^2)(a_3^2 + a_4^2)\{(p_1.p_4)(p_2.p_3) + (p_1.p_3)(p_2.p_4)\}$$

$$+ 4a_1 a_2 a_3 a_4 \{(p_1.p_4)(p_2.p_3) - (p_1.p_3)(p_2.p_4)\}] \ ,$$

$$|\mathcal{M}_2|^2 = 32|F_2|^2 [(a_1^2 + a_2^2)(a_3^2 + a_4^2)\{(p_1.p_4)(p_2.p_3) + (p_1.p_2)(p_3.p_4)\}$$

$$+ 4a_1 a_2 a_3 a_4 \{(p_1.p_4)(p_2.p_3) - (p_1.p_2)(p_3.p_4)\}] \ ,$$

$$Re(\mathcal{M}_1^* \mathcal{M}_2) = 32 Re(F_1^* F_2)\{(a_1^2 + a_2^2)(a_3^2 + a_4^2) + 4a_1 a_2 a_3 a_4\}(p_1.p_4)(p_2.p_3) \ . \tag{12}$$

The calculation will be performed in the centre of mass (CM) frame of the outgoing electron and positron, as denoted by the momenta p_3 and p_4 respectively. In this frame we have;

$$p_1 = (E_\mu, 0, 0, p) \ ,$$

$$p_2 = (E_e, 0, 0, p) \ ,$$

$$p_3 = (E'_e, 0, p' \sin\theta, p' \cos\theta) \ ,$$

$$p_4 = (E'_e, 0, -p' \sin\theta, -p' \cos\theta) \ , \tag{13}$$

with

$$p = \frac{\sqrt{\lambda(m_\mu^2, m_e^2, s)}}{2\sqrt{s}} \simeq \frac{m_\mu^2 - s}{2\sqrt{s}} \ ,$$

$$p' = \frac{1}{2}\sqrt{s - 4m_e^2} \simeq \frac{1}{2}\upsilon\sqrt{s} \ ,$$

$$E_\mu = \frac{m_\mu^2 - m_e^2 + s}{2\sqrt{s}} \simeq \frac{m_\mu^2 + s}{2\sqrt{s}} \ ,$$

$$E_e = \frac{m_\mu^2 - m_e^2 - s}{2\sqrt{s}} \simeq \frac{m_\mu^2 - s}{2\sqrt{s}} \ ,$$
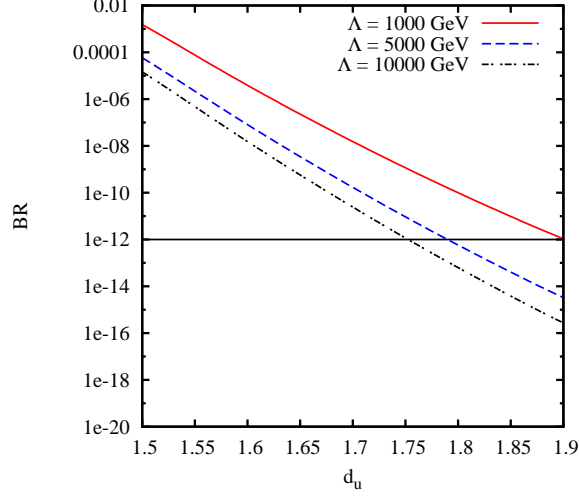
$$E'_e = \frac{\sqrt{s}}{2} \ , \tag{14}$$

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Figure 2: *Variation of the branching ratio with $d_{\mathcal{U}}$ for various values of $\Lambda_{\mathcal{U}}$. The other parameters are $a_1 = a_2 = 0.0001$ and $a_3 = a_4 = 0.01$*

and where $s = (p_1 - p_4)^2 = (p_2 + p_3)^2$, $v = \sqrt{1 - 4m_e^2/s}$. In this case $P^2 = m_\mu^2 + m_e^2 - 2E_\mu E_e + 2p^2$ and $Q^2 = m_\mu^2 + m_e^2 - 2E_\mu E_e' - 2pp' \cos\theta$.

$|\mathcal{M}|^2$, after summing over all the final states and averaging over the initial states, is then used to calculate the decay widths using the general expressions:

$$d\Gamma = \frac{1}{2E_\mu} \frac{d^3\overrightarrow{p}_2}{(2\pi)^3 2E_2} \frac{d^3\overrightarrow{p}_3}{(2\pi)^3 2E_3} \frac{d^3\overrightarrow{p}_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta(p_1 - p_2 - p_3 - p_4) \frac{1}{2}\frac{1}{2} |\mathcal{M}|^2 \ , \tag{15}$$

$$\frac{d\Gamma}{dsdx} = \frac{1}{2^9 \pi^3} \frac{1}{\sqrt{s}} \left(\frac{m_\mu^2}{s} - 1 \right) \sqrt{1 - \frac{4m_e^2}{s}} \frac{1}{2}\frac{1}{2} |\mathcal{M}|^2 \ , \tag{16}$$

where $x = cos(\theta)$ and θ is the scattering angle in the CM frame of the outgoing e^+e^-.

From equation (16) we calculate the branching ratio for the $\mu^- \to e^- e^+ e^-$ decay, where we note that the present experimental limits are [9]:

$$BR(\mu^- \to e^- e^+ e^-) < 1 \times 10^{-12} \ .$$

In figure (2) we present the variation of the branching ratio as a function of $d_{\mathcal{U}}$ for various values of $\Lambda_{\mathcal{U}}$. As we can observe from this figure the branching ratio is very sensitive to the scaling dimension of the unparticle $(d_{\mathcal{U}})$. In figure (3) we have shown the dependence of the branching ratio of $\mu^- \to e^- e^+ e^-$ on the various a_i's, where the coefficients a_1 and a_2 correspond to the lepton flavour violating (LFV) interactions. Again this has been done for different values of $\Lambda_{\mathcal{U}}$.

As can be seen from these plots the LFV process $\mu^- \to e^- e^+ e^-$ is very sensitive to $d_{\mathcal{U}}$ and the LFV couplings a_1 and a_2. The same LFV couplings will be involved in other LFV processes, such
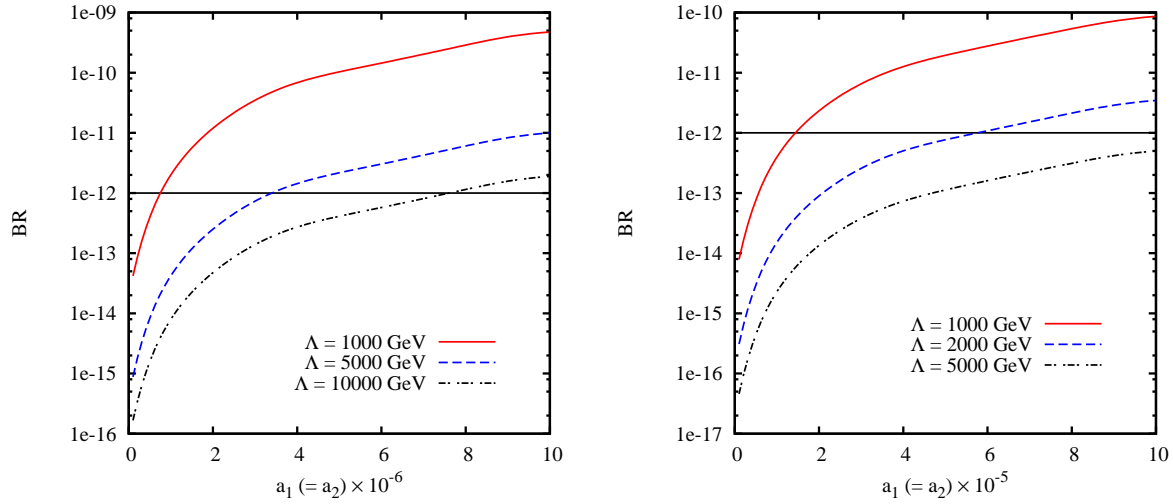
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Figure 3: *Variation of the branching ratio with $a_1(= a_2)$ for various values of $\Lambda_{\mathcal{U}}$. The other parameters are (a) Left panel: $a_3 = a_4 = 0.001$ and $d_{\mathcal{U}} = 1.6$, (b) Right panel: $a_3 = a_4 = 0.01$ and $d_{\mathcal{U}} = 1.8$.*

as $\mu \to e\gamma$. It would, therefore, be interesting to explore the phenomenology of LFV unparticle operators on radiative LFV processes and their possible coorelation with $\mu^- \to e^- e^+ e^-$. It is very well known that SUSY predicts a very strong coorelation in the LFV processes $\mu \to e\gamma$ and $\mu^- \to e^- e^+ e^-$, where these coorelations tend to change substantially in the Little Higgs model with T-parity [10]. As such, it would be interesting to study the coorelations amongst various LFV modes in the presence of a scale invariant sector/unparticles.

In conclusion, unparticle physics due to conformal invariance, might appear at the TeV scale and can lead to interesting phenomenological consequences which can be checked at low energy experiments. In this work we have studied the consequences of unparticle physics in the lepton flavour violating process $\mu^- \to e^- e^+ e^-$ and determined that the decay width is sensitive to the virtual effects of these unparticles.

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